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Introduction to Signed Brauer Algebra

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Abstract -In this paper we introduce signed Brauer algebra some of the basic definitions, lemmas and theorems and also introduce that signed brauer Algebra is semi simple.

Introduction

In 1937, Richard Brauer introduced the concept of Brauer Algebra. Brauer's algebra has a basis consisting of undirected graphs. In his paper a new class of algebra $\overrightarrow{D_f}$ is introduced namely signed brauer's algebra. The structure of these algebras $\overrightarrow{D_{f+1}}$ obtained by Wenzl [4]. The multiplication if these two graphs being the same as in $\overrightarrow{D_f}$ but each edge obtained in the multiplication in $\overrightarrow{D_f}$ is labelled in such a way to make $\overrightarrow{D_f}$ into an associative algebra. In this paper we will show is semi simple

1. SIGNED BRAUER'S ALGEBRA

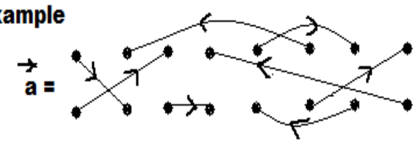
$\overrightarrow{D_f(x)}$ is defined over a field **K(x)** where **K** is any arbitrary field and **x** is an indeterminate.

A graph is said to be signed diagram if every edge is labelled by a plus or a minus sign and edges of a signed diagram are called signed edges. An edge labelled by a plus sign is called positive edge and an edge labelled by a minus sign is called negative edge.

A positive vertical edge will be denoted by $\downarrow$. A positive horizontal edge will be denoted by $\longrightarrow$, a negative vertical edge is denoted by $\uparrow$, and a negative horizontal edge will be denoted by $\longleftarrow$.

In other words, A brauer diagram with all its edges have + sign or – sign leads to a Signed brauer diagram. Let $\vec{v}_f$ be the set of all signed diagram with 2f vertices and f signed edges arranged in 2 lines, the connected components of such a diagram being a single signed diagram. The underlying any diagram is called signed brauer diagram whose edges are all positive is denoted by b. Let $\vec{D}_f$ be the vector space spanned by $\vec{v}_f$ over **K**.

Example



If $\vec{a}$ and $\vec{b}$ are two signed diagram then the new edge obtained in the product $\vec{ab}$ is labelled by a plus or minus sign according as the number of negative edges obtained from $\vec{a}$ and $\vec{b}$ to form this edge is even or odd respectively.

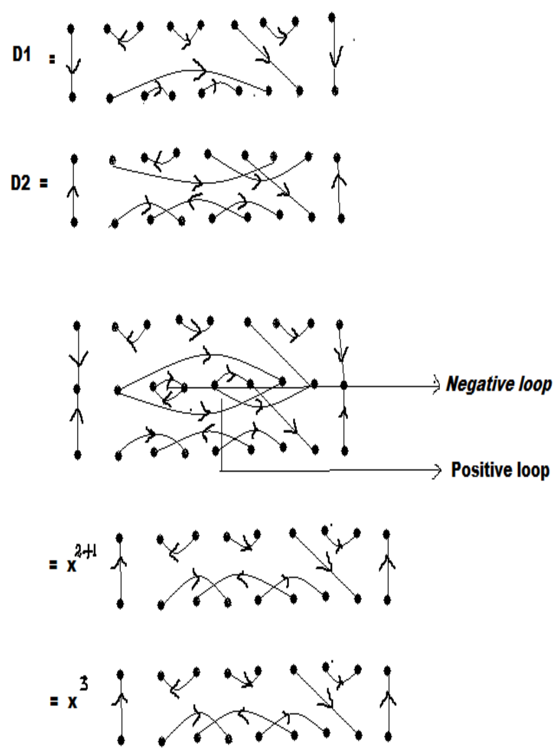
A loop β in $\vec{ab}$ is said to be positive if number of negative edges obtained from $\vec{a}$ and $\vec{b}$ to form this loop β is even.

A loop β in $\vec{ab}$ is said to be negative if number of negative edges obtained from $\vec{a}$ and $\vec{b}$ to form this loop is odd.

A positive loop β in $\vec{ab}$ is replaced by the variable x^2 in $\vec{ab}$ and a negative loop β in $\vec{ab}$ is replaced by the variable x in $\vec{ab}$.

Where D_1 is the number of positive loop in $\vec{ab}$ D_2 is the number of negative loop

Example



Lemma: 2.1

Let $\vec{a}$, $\vec{b}$, $\vec{c} \in v_f$ then $\left(\vec{a}\,\vec{b}\right)\vec{c} = \vec{a}\left(\vec{b}\,\vec{c}\right)$

Proof:

$\vec{a}$, $\vec{b}$, $\vec{c}$, are signed diagrams from $\vec{v}_f$

Definition 2.2:

Lower tower and upper lower:

Let $\vec{a}$, $\vec{b}$, $\vec{c} \in \vec{v}_f$ and α be a new edge formed in the product (ab) c and let $b_{1_{i_1}}$ $b_{1_{j_1}}$ be $b_{2_{i_2}}$ $b_{2_{j_2}}$ two consecutive vertical edges in α in the graph abc. Then the figure obtained in the brauer graph (ab)c by considering all the horizontal edges in $\vec{a}$ and $\vec{b}$, lying in between $b_{1_{i_1}}$ $b_{1_{j_i}}$, $b_{2_{i_2}}$ $b_{2_{k_2}}$ forming a part of α, is called a lower tower of α.

Proof of lemma:

By definition of multiplication of signed diagrams, it is clear that (ab) c = a(bc) where a, b, c are undirected graphs, so it is sufficient to prove that the signature of each new edge or a loop in $\left(\vec{a}\,\vec{b}\right)\vec{c}$ and the same. Let m_1, m_2, m_3 be number of negative edges respectively in $\vec{a}$, $\vec{b}$, $\vec{c}$ to form the new edge α or in loop β in $\left(\vec{a}\,\vec{b}\right)\vec{c}$

Let n be number of upper towers in the edge α or in loop β. Let n' be number of lower towers in the edge α or in loop β in figure a(bc). Let μ_i, 1 ≤ I ≤ n be number of negative edges in each upper tower. Let $\mu_i{}'$ be number of negative edges in each edge in α or in β in $\left(\vec{a}\,\vec{b}\right)\vec{c}$ after multiplying $\vec{a}\,\vec{b}$ with $\vec{c}$. Let λ_i $1 \le i \le n$ be number of negative in edges in each lower tower. Let λ_i' be number of negative edges in each edge α or in β in $\left(\vec{a}\,\vec{b}\right)\vec{c}$ after multiplying $\vec{b}$, $\vec{c}$ with $\vec{a}$.

Then $\mu_i \equiv \mu_i'$ (mod 2) where 1 ≤ i ≤ n

$\lambda_i \equiv \lambda_i'$ (mod 2) where 1 ≤ i ≤ n'

The number of negative edges in α in $\left(\vec{a}\,\vec{b}\right)\vec{c}$

$$= \Sigma \mu_i' + (m_1 + m_2) - \Sigma \mu_i + m_3$$

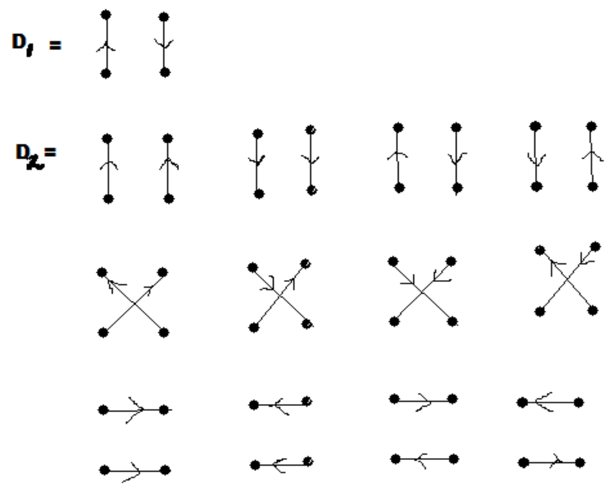
$$\equiv \Sigma \mu_i + (m_1 + m_2) - \Sigma \mu_i + m_3$$

$$\equiv m_1 + m_2 + m_3$$

similarly by, the number of negative edges in α in $\vec{a}\left(\vec{b}\,\vec{c}\right) \equiv m_1 + m_2 + m_3$

$$\therefore \left(\vec{a}\,\vec{b}\right)\vec{c} = \vec{a}\left(\vec{b}\,\vec{c}\right)$$

STRUCTURE OF SIGNED BRAUER ALGEBRA:



THEOREM: 2.4:

For signed Brauer algebra, the following hold.

i) $ei^2 = x^2 e_i$

ii) $e_i e_{i-1} e_i = e_i$

iii) $e_{i-1} e_i e_{i-1} = e_{i-1}$

iv) $g_i^{\,2} = 1$

v) $g_i\, \vec{h}_{i+1} = \vec{h}_i\, g_i$

vi) $\vec{h}_i^{\,2} = 1$ *for i = f also*

vii) $e_i h_i e_i = x e_i$

viii) $\overrightarrow{h_i} e_i = \overrightarrow{h_{i+1}}\, e_i$

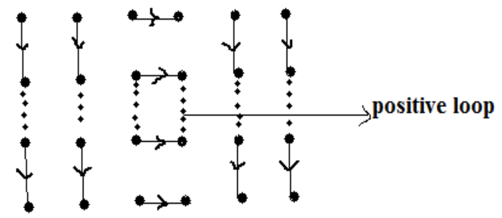
ix) $\overrightarrow{h_i}\, e_i = \overrightarrow{h_{i+1}} e_i$

x) $e_i \overrightarrow{h_i} = e_i \overrightarrow{h_{i+1}}$ where i = 1, 2,…f-1.

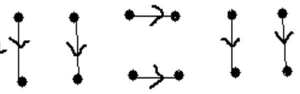
Proof:

i) To prove : $e_i^2 = x^2 e_i$

e_i^2 =



$= x^2$



$= x^2 e_i$

THEOREM: 2.5:

$\vec{S}_f \cong z_2 / s_f$ Where S_f is the symmetric group with f symbols and Z_2 is the group consisting of two elements

Proof:

The set of all graphs which do not contain any horizontal edge is denoted by $\overrightarrow{s_f}$

Define $\vartheta : \vec{s}_f \to \frac{z_2}{s_f} by \ \theta(\vec{b}) = (f, \pi^{-1}), \vec{b} \in \vec{s}_f$

`Where π is the underlying permutation of $\vec{b}$ in D_f and

f (i) = 0 if the vertical edge $(i, \pi(i))$ is positive

f (i) = 1 if the vertical edge $(i, \pi(i))$ is negative

[Since $z_2 = \{0,1\}$ are only 2 elements].

Claim: θ is isomorphism

i) Claim θ is 1-1

To prove $\theta\left(\vec{b}\right) = \theta\left(\vec{c}\right) \Rightarrow \vec{b} = \vec{c}$

i.e. $\theta\left(\vec{b}\right) = \theta\left(\vec{c}\right) \Rightarrow \left(f, T^{-1}\right) = \left(g, \pi^{-1}\right)$

$\Rightarrow f_\pi = g_\pi$ $\left[\left(f, \pi^{-1}\right) = f\pi\right]$ [By definition]

$\Rightarrow f.\pi^{-1} = g.\pi^{-1}$ [By definition $f\pi = f.\pi^{-1}$]

$\Rightarrow f.\pi^{-1} = g.\pi^{-1} = \bar{0} \in \frac{z_2}{s_f}$

$\Rightarrow (f-g)\pi^{-1} = \bar{0} \in \frac{z_2}{s_f}$

$\Rightarrow f - g = \bar{0}$

$\Rightarrow f = g$

$\Rightarrow \vec{b} = \bar{c} \therefore is\ 1-1$

ii) Claim : θ is onto

For every $\left(f, \pi^{-1}\right)$ in $\frac{z_2}{s_f} \exists \vec{b} \in \vec{s}_f \ni : \left(f, \pi^{-1}\right) = \theta\left(\vec{b}\right)$

θ is onto

iii) Claim : θ is homomorphism

$= \left(fg, \pi_1^{-1}, \pi_1^{-1} \pi_2^{-1}\right)$

$= \left(fg, \pi_1^{-1}, \left(\pi_2 \pi_1\right)^{-1}\right)$

$= \left(h, \left(\pi^1\right)^{-1}\right)$ where h= $fg, \pi_1^{-1}, \pi_2\pi_1 = \pi^{-1}$

$= \theta\left(\vec{P}_1 \ \vec{P}_2\right)$

Define: (f f') (i) = f (i) + f' (i) $\forall i \in$ { 1, 2….n}

n (i) = 0 $\Rightarrow$ f (i) = f (i) + g (π (i)) if f (i) = g (π (i))

n (i) = 1 $\Rightarrow$ h (i) = f (i) + g (π (i)) if f (i) $\neq$ g (π (i))

$$\left|\vec{s}_f\right| = \left|\frac{z_2}{s_f}\right| = 2f \ \ f!$$

[Since Z_2^n consists 2^n elements & S_f contains f symbols]

Hence θ is an isomorphism

CONCLUSION:

The extension of Brauer algebra, with positive and negative edges leads to signed brauer diagram which gave a new concept of signed brauer algebra. We proved signed brauer algebra $\vec{D}_f$ is semi simple. Even though it is non – commutative algebra it is semi simple.

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